It hence follows that the Saint-Venant-Mises equations utilized in this paper will be satisfied to within a small parameter m. This parameter will be substantial only at a significant distance from the body boundary. The transverse shock vanishes there and stresses on the zone boundaries are made continuous. However, the flow will be close to that considered near the body and the results obtained for the stresses and forces acting on the body can be used for an approximate estimate of the real quantities.

REFERENCES

1. LAVRENT'YEV M.A., Cumulative charge and the principles of its operation, Uspekhi Matem. Nauk, 12, 4, 1957.
2. GRIGORYAN S.S. A new law of friction and the mechanism of large-scale mining landslides and slumps, Dokl. Akad. Nauk SSSR, 244, 4, 1979.
3. KOCHIN N.E., KIBEL I.A. and ROZE N.V., Theoretical Hydrodynamics. 1, and 2, Fizmatgiz, Moscow, 1963.
4. SEDOV L.I., Mechanics of a Continuous Medium, 1 and 2, Nauka, Moscow, 1983.
5. FLITMAN L.M., On the boundary layer in certain problems of the dynamics of a plastic medium, Izv. Akad. Nauk SSSR, Mekhan. Tela, 1, 1982.
6. FLITMAN L.M., The boundary layer in the flow of a platic medium near a rough surface, PMM, 49, 4, 1985.
7. HILL R., The Mathematical Theory of Platicity, Gostekhizdat, Moscow, 1956.

Translated by M.D.F.

PMM U.S.S.R., Vol.54, No.4, pp.538-543, 1990
Printed in Great Britain

0021-8928/90 $10.00+0.00

THE THEORY OF THE FRACTURE OF A SUPERCONDUCTOR IN A MAGNETIC FIELD*

E.A. DEVYATKIN and I.V. SIMONOV

The stress-strain state of a superconductor in a static magnetic field is investigated from the point of view of the possibility of fracture. Only one force factor is taken into account, namely, the interaction between the field and the surface currents generated by the magnetic field (the Meissner effect /1/). When there are stress concentrators present (corner points, microcracks, inclusions etc.) comparatively weak magnetic fields, for which the specimen does not lose its property of ideal superconductivity, may turn out to be dangerous /1-3/. However, the formulation of the problem remains correct when the superconductor transforms into the normal phase (or simply for a normal conductor) in a variable intense magnetic field under skin-effect conditions and in a quasistatic mechanical state. In this case $t_1 \gg t_2$ is the condition of quasistatics, where t_1 and t_2 are the characteristic times of variation of the magnetic field and the range of wave deformation (volume or shape) of the characteristic dimension of the specimen. Moreover, when there are many factors present, this makes the problem a multiparametric one and extremely complicated to analyse, a preliminary investigation of the effect of each of these factors separately is advisable. The properties of the solutions of plane problems are analysed in detail, in particular, using the examples of regions of

**Prikl. Matem. Mekhan.*, 54, 4, 652-657, 1990

canonical forms: a corner, a cylinder, and a specimen with a cut in the form of an ellipse. The solution of a spatial problem for a sphere is given.

1. Formulation of the problem. An ideal superconductor (the region $\vartheta \in R^n$, $n = 2, 3$) surrounds a matrix of a normally conducting metal or vacuum. Each medium is uniform and isotropic in all its properties, linearly elastic for mechanical forces and has brittle strength. The region Ω (the supplement of $\bar{\vartheta}$ to R^n), where the matrix is contained, is penetrated by a magnetic field of strength **H** from sources in the same region and from currents induced on the surface of separation $\Gamma = \partial\vartheta$ for which the following physical conditions of ideal superconductivity are satisfied:

$$|H_\Gamma| < H_C(T), \quad T < T_C \tag{1.1}$$

where H_C and T_C are the critical values of the magnetic field and the temperature, and T is the temperature of the composite structure. The surface currents expel the magnetic field from the superconductors: $\mathbf{H} = 0$ in the region ϑ /1, 3/. For a superconductor of the second kind $H_C = H_{C_1}$, where H_{C_1} is the lower value of the critical field. The mixed state ($H_{C_1} < |\mathbf{H}_\Gamma| < H_{C_2}$), which is of undoubted interest for new composite superconductors /2/, is not considered here. However, the results obtained can also be used as an approximation for $|\mathbf{H}_\Gamma| < H_{C_1}$, so long as the field penetrates slightly into the region ϑ. Under skin-effect conditions /3/ limitations (1.1) are removed.

We will neglect the distribution of the field **H** and the currents in the depth of the region ϑ. This idealization is all the more correct the smaller the ratio h/L, where $h \lesssim 10^{-7}$ m is the depth of penetration of the magnetic field into the superconductor and L is the characteristic linear dimension of the region ϑ /1, 3/. We will not specify the sources of the magnetic field in any more detail except to say that the unperturbed field (when there is no superconductor) will be assumed to be known a priori.

The magnetic field strength can be represented in the form of the sum of the unperturbed field $\mathbf{H}_0$ and the field $\mathbf{H}_*$ of the superconductor. The perturbed field $\mathbf{H}_*$ is rotational everywhere in the region Ω and we can introduce a potential Φ. The result of the interaction between the field and the currents is a non-negative pressure p on the surface Γ from the side of the conductor (the tangential forces are equal to zero) /3/.

The mathematical formulation of problem 1 on determining the field **H**, the density of the surface currents $\mathbf{j}_\Gamma$ and the pressure p follows from Maxwell's equations, the conditions $\mathbf{j} = 0$ in the region Ω, $\mathbf{H} = 0$ in the region ϑ, the conditions of continuity of the normal component of the induction vector $\mu\mu_0\mathbf{H}$ on passing through the boundary Γ, the connection between the jump in the tangential component of the vector **H** and the currents $\mathbf{j}_\Gamma$ and the pressure /3/, the principle of superposition, and the conditions for the perturbation to disappear at infinity (if ϑ is a finite region), namely.

$$\Delta\Phi = 0, \ \mathbf{H}_* = \nabla\Phi\ (\Omega), \ \mathbf{H} = \mathbf{H}_0 + \mathbf{H}_* \tag{1.2}$$

$$\partial\Phi/\partial\rho = -H_{0\rho}(\Gamma), \ \Phi(\infty) = 0$$

$$\operatorname{rot}\mathbf{H} = \mathbf{j}_\Gamma\delta(\rho) \Rightarrow j_\Gamma = H_\tau; \quad p = {}^1/_2\mu\mu_0H_\tau^2 \tag{1.3}$$

Here ρ is the coordinate along the normal to the surface, Γ, $H_{0\rho}$, H_τ are the projections on the normal and on Γ, δ is the delta function and **H** is understood in the sense of generalized functions with carriers Γ and Ω /4/, μ is the magnetic permeability of the matrix, and μ_0 is the magnetic permeability of free space.

The first of Eqs.(1.3) serves to define the direction of the current, and the second (a consequence of it) gives the value of this current. For the potential Φ we single out the Neumann problem for Laplace's equation. The field **H**, the currents $\mathbf{j}_\Gamma$ and the pressure p can then be obtained from (1.2) and (1.3).

The formation of problem 1 is correct for a smooth surface Γ. If there are corner and conical points on Γ we must formulate additional conditions at these points /5/. They follow from considerations of the integrability of the energy density $w = {}^1/_2\mu\mu_0H^2$ and consist of the following inequalities for the indicator of the singularity of the solution as $r \to 0$ (here r is the distance to the singular point): $\Phi \sim r^\alpha, \alpha > 0$ is the corner point, and $\alpha > -{}^1/_2$ is a conical point.

For $n = 2$ the case of a single-component field $\mathbf{H} = (0, 0, H_z), H_z = H_z(x, y)$ must be considered separately. Then, in view of the corollary

$$\operatorname{rot}\mathbf{H} = 0 \Rightarrow \partial H_z/\partial x = \partial H_z/\partial y = 0\ (\Omega)$$

the solution of problem 1 can be written immediately

$$H_z = H = j_\Gamma = \text{const}, \quad p = {}^1/_2\mu\mu_0H^2 \tag{1.4}$$

Consequently, a constant current circulates in the surface Γ in the x, y plane and the pressure p is also constant.

After solving problem (1.2) and (1.3) we formulate problem 2. For a given system of volume and surface force factors, in which we also include the load p, and elastic parameters of the materials: E_ϑ, E_Ω are Young's moduli and $\nu_\vartheta, \nu_\Omega$ are Poisson's ratios, it is required to determine the stress state of the superconductor and the conductor.

Mathematically, problem 2 consists of solving the equations of the static theory of elasticity /6/ in the region ϑ and Ω for the following conditions for matching the solutions on the boundary of complete contact and the condition at infinity:

$$[\sigma_\rho] = p, \quad [\sigma_\tau] = [u] = 0, \quad \sigma(\infty) = \sigma_\infty$$

Here $[\sigma_\rho, \sigma_\tau, u]$ is the jump in the stress and displacement vectors on changing from the region ϑ to the region Ω, and σ is the stress tensor. It is assumed that one of the regions is finite, and that we are given the uniform stress state σ_∞ at infinity.

This problem belongs to a number of complex contact problems of the theory of elasticity. For bodies of canonical form (a cylinder, sphere, etc.) the solution can be constructed in the form of finite or infinite series in the separable parts of the solutions. Below, these solutions will be obtained neglecting the elastic resistance of the matrix.

The last step consists of analysing the stress field from the point of view of the brittle strength of materials. Using well-known ideas of the theory of fracture /7-10/ we will formulate one of the possible versions of the criterion of fracture, including three new parameters for each of the materials: a structural parameter or the radius of the nucleus of fracture r_C, and the strength for uniaxial extension and for pure shear, σ_0 and τ_0. The conditions for fracture on the interface, must, generally speaking, be stipulated separately, but the ahesion strength is often close to the strength of a weak material, which is also used here.

The analysis of the state of the system using this criterion is carried out as follows. The set of points M will be said to be susceptible to fracture if at these points local maxima of the tensile stresses σ or tangential stresses τ are reached (in space and with respect to the orientation of the area), such that $\max\sigma \geqslant \sigma_0$ or $\max\tau \geqslant \tau_0$. The set M also includes points of singularity of the stresses.

Around any point $m \in M$, such as around the centre, we will describe a spherical (R^3) or a cylindrical (R^2) surface Γ_C of radius r_C. Then the element of volume ω_C, inside this surface, is, by definition, in an elastic state if $\sigma_* = \max\sigma\,|_{\Gamma_C} < \sigma_0$ and $\tau_* = \max\tau\,|_{\Gamma_C} < \tau_0$, in a fractured state if $\sigma_* > \sigma_0$ or $\tau_* > \tau_0$, and in a limiting case if $\sigma_* = \sigma_0, \tau_* \leqslant \tau_0$ or $\sigma_* \leqslant \sigma_0, \tau_* = \tau_0$. A judgement can be made on the state of the system as a whole by adding up all the information on the state of the local regions ω_C. Thus, the state of the system is said to be limiting if at least one element ω_C is in such a state and there are no fractured elements, it is said to be in a fractured state if at least one element ω_C is in such a state and, finally, it is said to be in a state of elastic equilibrium if all the elements ω_C are elastic or the set M is empty. (We do not rule out other possible formulations of the criterion of fracture in view of the complexity of the phenomenon, the large variety of materials, and the absence of any single opinion on this problem).

In the case when $r_C \ll \lambda$, where λ is the characteristic dimension of the figure of the stress field around the point m, the analysis is simplified and is carried out for the values of σ and m at this point.

A study of the stress state is also important from the point of view of investigating the transition of a superconductor into the normal phase or into an intermediate state. As we know /1-3/, for certain superconductors it may be necessary to take into account the dependence of the critical magnetic field H_C on the stresses.

We will now investigate some specific examples. For simplicity, we will neglect the elastic resistance of the medium in the region Ω compared with the elastic resistance of the superconductor $(E_\Omega/E_\vartheta \ll 1$ or $E_\Omega = 0)$. The pressure p will be assumed to be the only load on the specimen.

2. ***The planar problem, the field*** $\boldsymbol{H = (0, 0, H_z)}$. The solution is given by formulas (1.4). Suppose the region ϑ is finite. Then the assertion holds that the cylindrical specimen is in a hydrostatic state and, even when there are corner points or initial cracks, will not be fractured for any values of the stress H_z (the most favourable case).

We will consider some cases when the region ϑ is external. Suppose that in the superconductor there is a circular opening, far from the boundary, and the field $(0, 0, H_z)$ is concentrated in it. Then, along the outline of the opening there will be tangential stretching forces of stress $\sigma = p$, and thus a fracture field of $H = \sqrt{2\sigma_0/(\mu\mu_0)}$.

For an ellipsoidal cut with semi-axes a, b $(a > b)$ in an unbounded superconductor, unloaded at infinity, susceptible points will be vertices with the last radius of curvature of the contour $r_* = b^2/a$.

We must distinguish the cases $r_* \ll r_C, r_* \sim r_C$ and $r_* \gg r_C$.

For $r_* \ll r_C$ (a thin ellipse) at distances from the vertex $\sim r_C$ one obtains an asymptotic form of Sneddon's solution /11/ for a cut. According to the criterion of fracture on the outline and using Sneddon's formula /11/, we can obtain an expression for the fracture field $H^2 = 2[\sigma_0/(\mu\mu_0)]\sqrt{2r_C/a}$.

For $r_* \gg r_C$ we obtain an estimate of the limiting state from the expression for the tensile stress at the vertex of the ellipse /11/ $H^2 = 2\sigma_0 b/(\mu\mu_0 a)$.

For the asymptotic transition $b/a \to 0$, $b^2/a \gg r_C \to 0$ we have $H \to 0$, i.e. for very small values of the parameter r_C the destructive field may be fairly small. An analytical estimate of the radius r_C is given in /8/ and the stress fields are obtained by comparison before the beginning of fracture for simple stretching of the specimen and around the vertex of the crack, which is in the limiting state. It follows from this that, for example, for metals $r_C \sim 10^{-3}-10^{-1}$ m, while different sources, where data are given on experimental measurements of this parameter, confirm this estimate.

For other forms of cuts, an answer can be obtained by analysing well-known solutions (for example, a lune and a thin cut or arbitrary form were considered in /11/).

3. *The planar problem, the field* $H = (H_x, H_y, 0)$. If the field of the vectors H are situated in the x, y plane, problem 1 is similar to a certain problem of non-cavitating flow past an underformed contour Γ by a laminar flow of an ideal incompressible liquid. The vector of the current density $\mathbf{j}_\Gamma$ is perpendicular to x, y plane. Non-zero compressive forces act almost everywhere on the contour Γ. In view of the non-uniformity of the pressure distribution tensile stresses may arise in the specimen, but, as previously, it can be said that for convex surfaces Γ tangential stresses will be the most dangerous. We will consider some examples of specimens of canonical forms.

Aperture angle α. The complex potentials

$$W_k = A_k z^{\beta_k+1}, \quad k = 1, 2 \quad (dW/dz = H_x - iH_y) \tag{3.1}$$
$$z = re^{i\theta}, \quad \beta_1 = \alpha/(2\pi - \alpha), \quad \beta_2 = (\alpha - \pi)/(2\pi - \alpha)$$

give solutions of the problems of "flow past a wedge by a flow H", symmetrical and antisymmetrical with respect to the bisector of the angle, from infinity (see the figure).

We will write expressions for the field and the pressure on the faces of the wedge

$$H_\tau = (\beta_k + 1) A_k r^{\beta_k}, \quad p = {}^1/_2\mu\mu_0(\beta_k + 1)^2 A_k{}^2 r^{2\beta_k} \tag{3.2}$$

The general expressions for the asymptotic forms of the functions H_τ and p of the corner point of an arbitrary contour Γ, as follows from the general theory of the behaviour of the solutions of elliptic equations at corner points of a region /5/, differe from the right-hand sides of Eqs.(3.2) by the terms $O(1)$, $O(r^{\beta_k+1})$, $r \to 0$.

We will analyse the signs of β_k: $\beta_1 \geqslant 0$ for all $0 \leqslant \alpha \leqslant 2\pi$, $\beta_2 \geqslant 0$ when $\pi \leqslant \alpha \leqslant 2\pi$ (the salient angle) and $\beta_2 \leqslant 0$ when $0 \leqslant \alpha \leqslant \pi$ (the re-entrant angle). Hence it follows that at the vertex of the re-entrant angle in the general case (the vector $\mathbf{H}_\infty$ is directed at an arbitrary angle to the bisector of the wedge) the field and the pressure are singular, and the stresses are also singular but compressive. The symmetrical case is an exception. The behaviour of the function H_Γ confirms the destruction of the superconductivity about the vertex of the corner; values of $H_\Gamma = H_C$ are achievable. Zones appear with the usual behaviour of the conductor in the region ϑ in which the magnetic field penetrates into the depth of the superconductor. This implies changes in the formulation of the problem and, possibly, in the asymptotic form of the solution at the corner (a problem that requires a separate consideration).

At the vertex of the salient angle the field and the pressure disappear as $r \to 0$, but inside the corner there are singular tensile stresses: $\sigma \sim Kr^{-\varkappa}$, $\varkappa > 0$. An estimate of the stress intensity factor K must be made starting from the solution of the problem as a whole.

A solution of one class of such problems - the flow past polygons is given on the basis of the Schwarz-Christoffel theorem /12/. Mechanical problem 2 can then also be solved using a conformal transformation of the region /13/. The dependence of the index $\varkappa$ on the angle

α is universal /14/ (whereas the coefficient K depends on the intensity of the sources and the geometry of the region). The limiting loads (with respect to the field) are found from the equation $K = \sigma_0 r_C^{\varkappa}$.

A cylinder in a transverse magnetic field. Using the solution of the problem of circulation-free flow of an ideal fluid past a cylinder /15/, we will determine the magnetic field strength and the pressure on the surface of the cylinder $r = 1$:

$$H_\tau = -2H_0 \sin\theta, \quad p = p_0 \sin^2\theta, \quad p_0 = 2\mu\mu_0 H_0^2$$

where θ is the angle between the vector $\mathbf{H}_0$ and the radius-vector $\mathbf{r}$, and p_0 is the maximum pressure. From the general solution of the first boundary-value problem of the theory of elasticity for a cylinder /6/ we can find the stress inside the body, normalized to p_0:

$$\sigma_{rr} = -\sin^2\theta, \quad \sigma_{\theta\theta} = {}^1/_2\,[(2r^2 - 1)\cos 2\theta - 1]$$
$$\sigma_{zz} = \nu\,(r^2 \cos 2\theta - 1), \quad \sigma_{r\theta} = {}^1/_2\,(r^2 - 1)\sin 2\theta$$

The normal stresses are only compressive, and hence fracture is possible due to the action of shear stresses. A maximum of the modulus of the stress $\sigma_{r\theta}$ is reached when $r = 0$, $\theta = \pm\pi/4$ and is equal to ${}^1/_2$. Other maximum shear stresses $({}^1/_2\,|\,\sigma_{rr} - \sigma_{zz}\,|,\ {}^1/_2\,|\,\sigma_{\theta\theta} - \sigma_{zz}\,|)$ are obtained for $\theta = \pm\pi/4$ on the surface of the cylinder, and they are the same and equal to ${}^1/_2 - \nu$. This is less than the maximum of the stress $\sigma_{r\theta}$.

We will calculate the limiting value of the field for fracture by shear: $H_p = \sqrt{\tau_0/(\mu\mu_0)}$. We can put forward the suggestion that when $H_0 > H_p$ a shear crack begins to develop from the centre of the specimen at angles $\theta = \pm\pi/4$.

4. The spatial problem of a sphere in a uniform magnetic field. By analogy with the solution of the corresponding hydrodynamic problem of the flow of an ideal fluid past a sphere /15/, we first obtain the values of the field and the pressure on the surface

$$H_\tau = -{}^3/_2 H_0 \sin\theta, \quad p = p_0 \sin^2\theta, \quad p_0 = {}^9/_8\,\mu\mu_0 H_0^2$$

The stress state of the elastic sphere can be obtained from the general solution /6/ (normalized to p_0)

$$\sigma_{rr} = {}^1/_3\eta\,\{[7 + \nu\,(2 + 3r^2)]\,(3\cos^2\theta - 1) - 2\eta^{-1}\}$$
$$\sigma_{\theta\theta} = -\eta\,\{2\nu + (7 + \nu)\,r^2 + [7 + 2\nu - 7\,(2 + \nu)\,r^2]\cos^2\theta\}$$
$$\sigma_{r\theta} = -{}^1/_2\eta\,(7 + 2\nu)\,(1 - r^2)\sin 2\theta$$
$$\sigma_{\varphi\varphi} = -\eta\,\{7 + 4\nu + [5\nu - (7 + 11\nu)\cos^2\theta]\,r^2\}, \quad \eta = (7 + 5\nu)^{-1}$$

The maximum shear stresses, as in the case of the cylinder, develop at the centre of the sphere in small areas $\theta = \pm\pi/4$ and are equal to

$$\max\,|\,\sigma_{r\theta}\,| = {}^1/_2\eta\,(7 + 2\nu) \approx {}^1/_2$$

On the surface of the sphere $(r = 1)$ we have

$$\sigma_{rr} = -\sin^2\theta, \quad \sigma_{\theta\theta} = 2\nu\eta - \sin^2\theta, \quad \sigma_{\varphi\varphi} = \eta\,[2\nu - (7 + 11\nu)\sin^2\theta]$$

The maximum tensile stresses (they are of the order of $0.2\,\nu$) are very much smaller than the maximum shear stresses. Hence, the qualitative and quantitative conclusions regarding the possible nature of the fracture and the value of the fracture field are similar to the case of a cylinder.

The authors thank R.V. Gol'dshtein for his comments.

REFERENCES

1. LINTON E.A., Superconductivity, Mir, Moscow, 1964.
2. GUREVICH A.V., MINTS R.G. and RAKHMANOV A.L., The Physics of Composite Superconductors, Nauka, Moscow, 1987.
3. LANDAU L.D. and LIFSHITZ E.M., Theoretical Physics. 8, Electrodynamics of Continuous Media, Nauka, Moscow, 1982.
4. GEL'FAND I.M. and SHILOV G.E., Generalized Functions and Actions on Them, Fizmatgiz, Moscow, 1959.
5. KONDRAT'YEV V.A., Boundary-value problems for elliptic equations in regions with conical and corner points, Trudy Mosk. O-va, 16, 1967.
6. NOWACKI W. , Theory of Elasticity, Mir, Moscow, 1975.
7. SIH G.C., Strain-energy-density factor applied to mixed mode crack problems, Intern. J. Fract. Mech. 10, 3, 1974.

8. SIH G.C., Prediction of crack growth under mixed mode conditions, Proc. Ist. USA-Greece Symp. on Mixed Mode Crack Propagation. Alphen aan den Rijn: Sijthoff; Nordhoff, 1981.
9. PARTON V.Z. and MOROZOV E.M., Mechanics of Elastoplastic Fracture, Nauka, Moscow, 1985.
10. CHEREPANOV G.P., Mechanics of Elastic Fracture, Nauka, Moscow, 1974.
11. MOROZOV N.F., Mathematical Problems of the Theory of Cracks, Nauka, Moscow, 1984.
12. LAVRENT'YEV M.A. and SHABAT B.V., Methods of the Theory of Functions of a Complex Variable, Nauka, Moscow, 1965.
13. MUSKHELISHVILI N.I., Some Fundamental Problems of the Mathematical Theory of Elasticity, Nauka, Moscow, 1966.
14. WILLIAMS M.L., Stress singularities resulting from various boundary conditions in angular corners of plates in extension, J. Appl. Mech. 19, 4, 1952.
15. LOITSYANSKII L.G., Mechanics of Liquids and Gases, Fizmatgiz, Moscow, 1959.

Translated by R.C.G.

PMM U.S.S.R., Vol.54, No.4, pp.543-549, 1990
Printed in Great Britain

0021-8928/90 $10.00+0.00

VARIATIONAL PRINCIPLES OF NON-LINEAR THEORY OF BRITTLE FRACTURE*

LE KHAN CHAU

An energy criterion of equilibrium of a non-linearly elastic body with a crack is formulated. Equations of statics and conditions which must hold at the outer boundary of the body, at its surface and at the slit edge, are derived. An evolutionary variational inequality is postulated, from which the formulation of the dynamic problem of the motion of a body with an expanding crack follow.

1. *Forumulation of the problem.* Let us consider an elastic solid which has a defect when in its natural state. The defect can be modelled by a displacement discontinuity surface, which will be called, from now on, the crack. Let this crack be situated on a smooth, two-dimensional surface Ω, with a smooth boundary $\partial\Omega$. We take the natural configuration of the body occupying the region $V_\Omega = V \setminus (\Omega \cup \partial\Omega)$ of three-dimensional Euclidean space as the reference configuration, and denote the Cartesian coordinates of the particles of the body in this configuration by X_a, $a = 1, 2, 3$. In the deformed state the Cartesian coordinates of the particles will be given by the formulas

$$x_i = x_i(X_1, X_2, X_3), \quad i = 1, 2, 3$$

The coordinates x_i fill the volume v of the current configuration. If the deformed body with a crack is in a state of equilibrium, the functions $x_i(X_a)$ will map in 1:1 correspondence with a positive Jacobian. When X_a pass through Ω, the functions x_i become discontinuous. The traces $x_i(X_a)$ on both sides of Ω describe the surfaces of the crack in the deformed state (Fig.1).

The first problem consists of establishing the criterion of equilibrium of the configuration $x_i(X_a)$. With this purpose in mind, we shall formulate the following variational principle: in order for the defromed body with a crack to remain in equilibrium, it is necessary and sufficient that the variation in the energy of the body taken in a specified configuration be greater than, or equal to zero for all admissible configurations. We shall call a virtual configuration of the body admissible, if its displacement discontinuity surface contains Ω, or if it coincides with it.

If the body has no crack, the criterion of equilibrium in the class of all continuous configurations reduces to the well-known principle of stationarity of the energy of a non-linearly elastic body /1-3/. The generality of the energy criterion of equilibrium was satisfactorily demonstrated for other mechanical systems by Gibbs /4/. The papers by Griffiths'

**Prikl.Matem.Mekhan.*, 54, 4, 658-665, 1990